3

Duties of induction motors

Contents

3.1 Duty cycles 3/65
3.2 Continuous duty (CMR) \((S_1)\) 3/65
3.3 Periodic duties 3/65
   3.3.1 Short-time duty \((S_2)\) 3/65
   3.3.2 Intermittent periodic duty \((S_3)\) 3/65
   3.3.3 Intermittent periodic duty with start \((S_4)\) 3/65
   3.3.4 Intermittent periodic duty with start and brake \((S_5)\) 3/66
   3.3.5 Continuous duty with intermittent periodic loading \((S_6)\) 3/67
   3.3.6 Continuous duty with start and brake \((S_7)\) 3/67
   3.3.7 Continuous duty with periodic speed changes \((S_8)\) 3/68
   3.3.8 Non-periodic duty \((S_9)\) 3/69
   3.3.9 Duty with discrete constant loads \((S_{10})\) 3/69
3.4 Factor of inertia \((FI)\) 3/70
3.5 Heating and cooling characteristic curves 3/70
   3.5.1 Time constants 3/70
   3.5.2 Heating curves 3/71
   3.5.3 Cooling curves 3/72
3.6 Drawing the thermal curves 3/73
   3.6.1 From cold conditions 3/73
   3.6.2 From hot conditions 3/75
3.7 Rating of short-time motors 3/77
3.8 Equivalent output of short-time duties 3/77
3.9 Shock loading and use of a flywheel 3/79
   3.9.1 Size of flywheel 3/80
   3.9.2 Energy stored by the flywheel 3/80

Relevant Standards 3/81
List of formulae used 3/81
3.1 Duty cycles

Unless otherwise specified, the rating of the motor will be regarded as its continuous maximum rating (CMR), defined by duty $S_1$ as noted below. But a machine is not always required to operate at a constant load. Sometimes it must operate at varying loads, with a sequence of identical operations, involving starts, stops braking, speed control and reversals, with intermittent idle running and de-energized periods etc. (e.g. a hoist, a crane, a lift or other applications). Using a CMR motor for such applications, with a rating corresponding to the maximum short-time loading will mean an idle capacity during no-load running or de-energized periods and a constant drain on energy, in addition to a higher cost of installation. To economize on the size of machine for such applications, IEC 60034-1 has defined a few duty cycles, as noted briefly below. These may be considered while selecting an economical size of machine and yet meet the variable load demands safely. Such motors may be running over-loaded during actual loading but for shorter durations not sufficient to exceed the permissible temperature rise limits. They dissipate excessive heat during idle running or de-energized periods to reach a thermal equilibrium at the end of the load cycle. These duties are described in the following sections.

3.2 Continuous duty (CMR) ($S_1$)

The operation of a motor at a rated load may be for an unlimited period to reach thermal equilibrium (Figure 3.1) and possible applications are pumps, blowers, fans and compressors.

3.3 Periodic duties

3.3.1 Short-time duty ($S_2$)

In this case the operation of the motor is at a constant load during a given time just adequate to attain the maximum permissible temperature rise, followed by a rest and de-energized periods of long durations to re-establish equality of motor temperature with the cooling medium (Figure 3.2). The motor should restart for the next cycle only when it has attained its ambient condition. The recommended values for short-time duty are 10, 30, 60 and 90 minutes. The type designation for a particular rating, say for 30 minutes, will be specified as $S_2 = 30$ minutes. Likely applications are operation of lock gates, sirens, windlasses (hoisting) and capstans.

3.3.2 Intermittent periodic duty ($S_3$)

This is a sequence of identical duty cycles, each consisting of a period of operation at constant load and a rest and de-energized periods. The period of energization may attain the maximum permissible temperature rise ($\theta_m$). The period of rest and de-energization is sufficient to attain thermal equilibrium during each duty cycle (Figure 3.3). In this duty the starting current $I_{st}$ does not significantly affect the temperature rise. Unless otherwise specified, the duration of each duty cycle should be 10 minutes. The recommended values for the cyclic duration factor $CDF$ are 15%, 25%, 40% and 60%. The type designation for a particular rating, say for 40%, will be specified as $S_3 = 40\%$.

Cyclic duration factor $CDF = \frac{N}{N+R}$

where

$N =$ operation under rated conditions

$R =$ at rest and de-energized and

$\theta_l =$ temperature rise reached during one duty cycle ($\theta = 0$)

Likely applications are valve actuators and wire drawing machines.

3.3.3 Intermittent periodic duty with start ($S_4$)

This is a sequence of identical duty cycles, each consisting of
of a period of start, a period of operation at constant load and a rest and de-energized periods. The starting, operating, rest and de-energized periods are just adequate to attain thermal equilibrium during one duty cycle (Figure 3.4). In this duty the motor is stopped, either by natural deceleration, after it has been disconnected from the supply source, or by mechanical brakes, which do not cause additional heating to the windings:

$$CDF = \frac{D + N}{D + N + R}$$

where

- $D =$ period of starting
- $N =$ operation under rated conditions
- $R =$ at rest and de-energized,
- $\theta_1 =$ temperature rise reached during one duty cycle ($\approx 0$)

For this duty cycle, the abbreviation is followed by the indication of the cyclic duration factor, the number of duty cycles per hour (c/h) and the factor of inertia ($FI$). (See Section 3.4 for $FI$.) Thus, for a 40% $CDF$ with 90 operating cycles per hour and factor of inertia of 2.5, the cycle will be represented by

$S_4 - 40\% - 90 \text{ c/h and } FI - 2.5$

Likely applications are hoists, cranes and lifts.

### 3.3.4 Intermittent periodic duty with start and brake ($S_5$)

This is a sequence of identical duty cycles, each consisting of a period of start, a period of operation at constant load, a period of braking and a rest and de-energized periods. The starting, operating, braking, rest and de-energized periods are just adequate to attain thermal equilibrium during one duty cycle (Figure 3.5). In this duty braking is rapid and is carried out electrically:

$$CDF = \frac{D + N + F}{D + N + F + R}$$

where

- $D =$ period of starting
- $N =$ operation under rated conditions
- $F =$ electric braking
- $R =$ at rest and de-energized, and
- $\theta_1 =$ temperature rise attained during one duty cycle ($\approx 0$)

For this duty cycle also, the abbreviation is to be followed by the indication of the $CDF$, the number of duty cycles per hour (c/h) and the $FI$, e.g.

$S_5 - 40\% - 90 \text{ c/h and } FI - 2.5$

Likely applications are hoists, cranes and rolling mills.
3.3.5 Continuous duty with intermittent periodic loading ($S_6$)

This is a sequence of identical duty cycles, each consisting of a period of operation at constant load and a period of operation at no-load. The repeat load and no-load periods are just adequate to attain thermal equilibrium during one duty cycle. There is no rest and de-energizing period,

![Diagram of Continuous Duty with Intermittent Periodic Loading ($S_6$)](image)

(Figure 3.6). Unless otherwise specified, the duration of the duty cycle will be 10 minutes.

The recommended values of $CDF$ are 15%, 25%, 40% and 60%:

$$CDF = \frac{N}{N + V}$$

where

- $N$ = operation under rated conditions
- $V$ = operation on no-load and
- $\theta_t$ = temperature rise attained during one duty cycle corresponding to the no-load heating

The designation in this case will be expressed as $S_6 - 40\% CDF$

Likely applications are conveyor belts and machine tools.

3.3.6 Continuous duty with start and brake ($S_7$)

This is a sequence of identical duty cycles, each consisting of a period of start, a period of operation at constant load and a period of electric braking. The start, operating and braking periods are just adequate to attain thermal equilibrium during one duty cycle. There is no rest and de-energizing periods (Figure 3.7):

$$CDF = \frac{D + N + F}{D + N + F} = 1$$

where

- $D$ = period of starting
- $N$ = operation under rated conditions

![Diagram of Continuous Duty with Starts and Brakes ($S_7$)](image)
$F$ = electric braking and
$\theta_t$ = temperature rise attained during one duty cycle
= corresponding to the no-load heating.

For this duty cycle also, the abbreviation is followed by the indication of number of cycles per hour and the $FI$.
For example, for 300 c/h and $FI$ 2.5

$S7 – 300 \text{ c/h } FI – 2.5$

Likely applications are machine tools.

### 3.3.7 Continuous duty with periodic speed changes ($S_8$)

This is a sequence of identical duty cycles, each consisting of a period of operation at constant load, corresponding to a determined speed of rotation, followed immediately by a period of operation at another load, corresponding to another speed of rotation, say, by change of number of poles. The operating periods are just adequate to attain thermal equilibrium during one duty cycle. There is no rest and de-energizing period (Figure 3.8):

\[
CDF = \frac{D + N_1}{D + N_1 + F_1 + N_2 + F_2 + N_3}
\]

at speed $Nr_1$ for load $P_1$

and

\[
= \frac{F_1 + N_2}{D + N_1 + F_1 + N_2 + F_2 + N_3}
\]

at speed $Nr_2$ for load $P_2$

and

\[
= \frac{F_2 + N_3}{D + N_1 + F_1 + N_2 + F_2 + N_3}
\]

at speed $Nr_3$ for load $P_3$

where

$F_1, F_2$ = changeover of speed by acceleration.
$D$ = electrical braking, $Nr_3$ to $Nr_1$
$N_1, N_2, N_3$ = operation under rated conditions and
$\theta_t$ = temperature reached during one duty cycle
= corresponding to the heating under rated conditions ($P_1$ as in Figure 3.8)

**Figure 3.8** Continuous duty with periodic speed changes, $S_8$
We have considered three different speeds (lower $N_r_1$ to higher $N_r_3$) for this duty cycle, having three $CDF$s for one cycle, each corresponding to a different speed.

For this duty type, the abbreviation is followed by the indication of the number of duty cycles per hour, the $FI$ and the load at the various speeds.

As an illustration the $CDF$ must be indicated for each speed as follows:

<table>
<thead>
<tr>
<th>$c/h$</th>
<th>$FI$</th>
<th>$kW$</th>
<th>Speed (r.p.m.)</th>
<th>$CDF$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2.5</td>
<td>10</td>
<td>1440</td>
<td>60</td>
</tr>
<tr>
<td>20</td>
<td>2.5</td>
<td>6</td>
<td>960</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>2.5</td>
<td>4</td>
<td>730</td>
<td>40</td>
</tr>
</tbody>
</table>

Likely applications are where the motor is required to run at different speeds.

### 3.3.8 Non-periodic duty ($S_9$)

This is a type of duty in which load and speed both vary non-periodically, unlike the periodic duty cycles noted above. The motor now supplies variable load demands at varying speeds and varying over-loads, but within the permissible temperature rise limits. It is a duty similar to duty cycle $S_8$, except that sometimes the over-loads may exceed the full load but are within the thermal withstand limit of the motor (Figure 3.9):

\[
D = \text{period of starting}
\]
\[
N_1, N_2, N_3 = \text{operations within rated load (} P_1 \text{)}
\]
\[
N_4 = \text{operation during over-load (} P_2 \text{)}
\]
\[
F = \text{changeover of speed by electrical braking}
\]
\[
R = \text{at rest and de-energized,}
\]
\[
\theta_t = \text{temperature rise reached during one duty cycle (} = 0 \text{)}
\]

#### 3.3.9 Duty with discrete constant loads ($S_{10}$)

This is a type of duty consisting of a number of varying loads, not more than four in each cycle. Each load is performed for sufficient duration to allow the machine to attain its thermal equilibrium (Figure 3.10). It is, however, permitted that each load cycle may not be identical, provided that each discrete loading during one particular load cycle is performed for a sufficient duration to attain thermal equilibrium. The temperature attained during each discrete loading is within permissible limits or within such limits that if it exceeds the permissible limit, the thermal life expectancy of the machine is not affected. For example, performing one discrete loading $P_2$, as in Figure 3.10, the temperature reached ($\theta_2$) may exceed the permissible limit ($\theta_m$) for a short duration ($t_2$), but the final temperature at the end of the cycle is still such that the next duty cycle can be performed. The short duration excess temperature, $\theta_2$, reached while performing the load duty $P_2$, will not however, be detrimental to the thermal life expectancy of the machine. Referring to Figure 3.10,

![Figure 3.9 Duty with non-periodic load and speed variations, $S_9$](image-url)
3.4 Factor of inertia (FI)

This is the ratio of the total moment of inertia referred to the motor shaft to the moment of inertia of the motor. If the motor moment of inertia is \( GD_M^2 \) and the load moment of inertia at motor speed, \( GD_L^2 \), then

\[
FI = \frac{GD_M^2 + GD_L^2}{GD_M^2} \quad (3.1)
\]

\( (GD^2 \text{ values are weight moments of inertia}) \)

### Example 3.1

If \( GD_M^2 = 0.16 \text{ kg m}^2 \)

and \( GD_L^2 = 0.8 \text{ kg m}^2 \) at motor speed

then \( FI = \frac{0.16 + 0.8}{0.16} = 6 \)

3.5 Heating and cooling characteristic curves

The heating and cooling behaviour of an induction motor, up to around twice the rated current, may be considered as exponential, as a part of the heat generated is offset by the heat sink (heat dissipation) through the windings. But beyond \( 2I_r \) it should be considered adiabatic (linear), as the heat generated is now quick and the winding insulation may not be able to dissipate this heat equally quickly, when it occurs for a short duration. Since a motor would normally operate at around \( I_r \) except during abnormal operating conditions, the exponential heating and cooling characteristics are more relevant during a normal run. They determine the performance of a motor, particularly when it is required to perform intermittent duties, and help determine safe loading, starts and brakings etc. (See curves (a) and (b) of Figure 3.11). They also assist in providing a thermal replica protection to large motors. With the help of these curves a motor protection relay (Section 12.5) can be set to closely monitor the thermal conditions prevailing within the machine, and provide an alarm or trip when the operating temperature exceeds the safe boundaries. These curves are known as thermal withstand curves and are provided with the motors as a standard practice by motor manufacturers. But when these curves are not available at a site and a thermal, IDMT or a motor protection relay (Chapter 12) is required to be set during commissioning, then the procedure described in Section 3.6 can be adopted to establish them. To determine them it is, however, essential to know the heating and cooling time constants of the motor, which are provided by the motor manufacturer.

3.5.1 Time constants

These are the times in which the temperature rises or falls by 0.632 times its maximum value \( \theta_m \) and are provided by the machine manufacturer. They are also shown in Figure 3.11.

**Significance of thermal time constants**

The short time rating of a CMR motor varies with its thermal time constant and may differ from one manufacturer to another depending upon the cooling design adapted and its effectiveness. The shorter the thermal constant, the lower will be the short time rating CMR motor can perform. It is not, however, practical to achieve the thermal time constant infinitely high, which is a compromise with the economics of the motor’s design such as size, wall thickness of the housing, number and depth of cooling fins and efficiency of the cooling fan.
3.5.2 Heating curves

Exponential heating on a cold start

The temperature rise corresponding to the rated current of the machine can be expressed exponentially by

\[ q_c = q_m (1 - e^{-t/\tau}) \]  

(3.2)

where

- \( q_c \): temperature rise of the machine on a cold start above the ambient temperature, after \( t \) hours (°C)
- \( q_e \): end temperature of the machine in °C after time \( t \) and \( \theta_a \) the ambient temperature in °C

\[ q_c = q_e - q_a \]

\( q_m \): steady-state temperature rise or the maximum permissible temperature rise of the machine under continuous operation at full load in °C, e.g. for a class B motor, operating continuously in a surrounding medium with an ambient temperature of 45°C

\[ q_m = 120 - 45 = 75°C \]  

(Table 9.1)

\[ \theta_m = 120 - 45 = 75°C \]

Note

For intermittent temperature rises between \( \theta_c \) and \( \theta_m \) as applicable to curves (c) and (d) of Figure 3.11, \( \theta_m \) may be substituted by the actual temperature on the heating or cooling curves.

\( t \) = time of heating or tripping of the relay (hours)

\( \tau \) = heating or thermal time constant (hours). The larger the machine, the higher this will be and vary from one design to another. It may fall to a low of 0.7–0.8 hour.

The temperature rise is a function of the operating current and varies in a square proportion of the current. The above equation can therefore be more appropriately written in terms of the operating current as

\[ K \cdot I_1^2 = I_t^2 (1 - e^{-\tau/t}) \]  

(3.3)

where

- \( I_t \): rated current of the motor in A and
- \( K \): a factor that would depend upon the type of relay and is provided by the relay manufacturer. Likely values may be in the range of 1 to 1.2

\( I_1 \): actual current the motor may be drawing

Test check

(i) For rated current at \( t = 0 \)

\[ \theta_c = \theta_e - \theta_a = I_t^2 (1 - e^{-0}) \]

\[ = 0, (e^0 = 1) \]

or \( \theta_c = \theta_a \), which is true

(ii) At \( t = \infty \)

\[ \theta_c = \theta_m (1 - e^{-\infty}) \]

\[ = \theta_m (e^{-\infty} = 0) \]

which is also true.

The relative temperature rise in a period \( t \) after the operating current has changed from \( I_0 \) to \( I_1 \)

\[ \theta_c \text{ (relative)} = (I_1^2 - I_0^2) (1 - e^{-\tau/t}) \]

If \( I_1 \) is higher than \( I_0 \), it will be positive and will suggest

![Figure 3.11 Heating and cooling curves](image-url)
a temperature rise. If $I_1$ is lower than $I_0$ then it will be negative and will suggest a temperature reduction.

**Exponential heating on a hot start**

This can be expressed by

$$\theta_h = \theta_0 + (\theta_1 - \theta_0)(1 - e^{-t/\tau})$$

and in terms of operating current

$$\theta_h = I_0^2 + (I_1^2 - I_0^2)(1 - e^{-t/\tau}) \quad (3.4)$$

where

$\theta_h =$ temperature rise of the machine on a hot start above the ambient temperature, after $t$ hours in $^\circ$C

$\theta_0 =$ initial temperature rise of the machine above the ambient in $^\circ$C

$\theta_1 =$ end temperature rise of the machine above the ambient in $^\circ$C

$\theta_i =$ initial temperature of the hot machine in $^\circ$C before a restart

$I_0 =$ initial current at which the machine is operating

$I_1 =$ actual current the motor may be drawing

Hence Equation (3.4) can be rewritten as

$$K \cdot I_i^2 = I_0^2 + (I_1^2 - I_0^2)(1 - e^{-t/\tau})$$

For the purpose of protection, $t$ can now be considered as the time the machine can be allowed to operate at a higher current, $I_1$, before a trip

$\therefore \: t = \text{tripping time.}$

Simplifying the above,

$$e^{-t/\tau} = \frac{I_1^2 - kI_i^2}{I_1^2 - I_0^2}$$

or

$$e^{t/\tau} = \frac{I_1^2 - I_0^2}{I_1^2 - kI_i^2}$$

and

$$t = \tau \log_e \frac{I_1^2 - I_0^2}{I_1^2 - kI_i^2} \quad (3.5)$$

With the help of this equation the thermal curve of a machine can be drawn on a log-log graph for a known $\tau$, $t$ versus $I_i/I_1$ for different conditions of motor heating prior to a trip (Figure 3.12). The relay can be set for the most appropriate thermal curve, after assessing the motor’s actual operating conditions and hence achieving a true thermal replica protection.

Equations (3.2) to (3.5) are applicable only when the heating or cooling process is exponential, which is true up to almost twice the rated current as noted above. Beyond this the heating can be considered as adiabatic (linear). At higher operating currents the ratio $t/\tau$ diminishes, obviously so, since the withstand time of the motor reduces sharply as the operating current rises. At currents higher than $2I_r$, the above formulae can be modified as below.

**Adiabatic heating on a cold start**

$$\theta_c = \theta_e - \theta_a = I_1^2 \cdot \frac{t}{\tau} \quad (3.6)$$

**Adiabatic heating on a hot start**

$$\theta_h = \theta_e - \theta_a = \theta_0 + (\theta_1 - \theta_0)t/\tau$$

or

$$= I_0^2 + (I_1^2 - I_0^2)t/\tau \quad (3.7)$$

**3.5.3 Cooling curves**

The residual temperature fall in terms of time, after the motor current is reduced to zero, can be expressed exponentially by

$$\theta = \theta_m \cdot e^{-t/\tau'}$$ \quad (3.8)

where

$\tau' =$ cooling time constant in hours. It is higher than the heating time constant $\tau$. When the machine stops, its cooling system also ceases to function, except for natural cooling by radiation and convection. The machine therefore takes a longer time to cool than it does to heat.
3.6 Drawing the thermal curves

These can be drawn for temperature versus time or current versus time as desired, depending upon the calibration of the device, such as a motor protection relay. Below we provide a brief procedure to draw these curves.

3.6.1 From cold conditions

(a) For \( I_1 \leq 200\% I_r \)

\[
\frac{\theta_c}{\theta_m} = \left( \frac{I_1}{I_r} \right)^2 \cdot \left( 1 - e^{-t/\tau} \right)
\]

or

\[
\frac{\theta_c}{\theta_m} = \left( \frac{I_1}{I_r} \right)^2 \left( 1 - \frac{1}{e^{t/\tau}} \right)
\]

(b) For \( I_1 > 200\% \)

\[
\frac{\theta_c}{\theta_m} = \left( \frac{I_1}{I_r} \right)^2 \cdot \frac{t}{\tau}
\]

where \( \theta_m \) and \( \tau \) are design parameters and are provided by the machine manufacturer. The curves can now be plotted in the following ways.

For thermal settings

\( \theta_c/\theta_m \) versus \( t/\tau \) different ratios of \( (I_1/I_r) \) as shown in Table 3.1(a) and Figure 3.13(a), for \( I_1/I_r \leq 200\% \), and Table 3.1(b) and Figure 3.13(b) for \( I_1/I_r > 200\% \). Since \( \theta_m \) and \( \tau \) are known, \( t \) can be calculated for different \( \theta_c \)’s corresponding to different motor currents. The relay can then be set to provide a thermal replica protection.

Note: For ease of illustration two graphs are drawn \( \left\{ \frac{I_1}{I_r} \leq 200\% \text{ and } \frac{I_1}{I_r} > 200\% \right\} \). For actual use, the relevant portions of the graphs (as marked) alone must be drawn on one common graph. More points can be plotted in the required region for a closer setting of the relay.

Figure 3.13 Thermal curves to set the relay for over-current protection corresponding to different operating temperatures.
Table 3.1  In terms of thermal settings \( \frac{\theta_C}{\theta_m} \) versus \( \frac{I_l}{I_t} \)

(a) For \( \frac{I_l}{I_t} \leq 200\% \)

<table>
<thead>
<tr>
<th>( t/\tau )</th>
<th>( 1 - \frac{1}{e^{t/\tau}} )</th>
<th>( \frac{\theta_C}{\theta_m} = \left( \frac{I_l}{I_t} \right)^2 \times \left( 1 - \frac{1}{e^{t/\tau}} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.632</td>
<td>( I_l/I_t = 0.5 ) ( I_l/I_t = 0.75 ) ( I_l/I_t = 1.0 ) ( I_l/I_t = 1.25 ) ( I_l/I_t = 1.5 ) ( I_l/I_t = 2.0 )</td>
</tr>
<tr>
<td>2</td>
<td>0.865</td>
<td>( I_l/I_t = 0.5 ) ( I_l/I_t = 0.75 ) ( I_l/I_t = 1.0 ) ( I_l/I_t = 1.25 ) ( I_l/I_t = 1.5 ) ( I_l/I_t = 2.0 )</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>( I_l/I_t = 0.5 ) ( I_l/I_t = 0.75 ) ( I_l/I_t = 1.0 ) ( I_l/I_t = 1.25 ) ( I_l/I_t = 1.5 ) ( I_l/I_t = 2.0 )</td>
</tr>
<tr>
<td>4</td>
<td>0.982</td>
<td>( I_l/I_t = 0.5 ) ( I_l/I_t = 0.75 ) ( I_l/I_t = 1.0 ) ( I_l/I_t = 1.25 ) ( I_l/I_t = 1.5 ) ( I_l/I_t = 2.0 )</td>
</tr>
<tr>
<td>5</td>
<td>0.993</td>
<td>( I_l/I_t = 0.5 ) ( I_l/I_t = 0.75 ) ( I_l/I_t = 1.0 ) ( I_l/I_t = 1.25 ) ( I_l/I_t = 1.5 ) ( I_l/I_t = 2.0 )</td>
</tr>
</tbody>
</table>

These high values of \( \theta_C/\theta_m \) are not permissible.
Plot curves for \( t/\tau < 1 \) also for settings at higher currents.

(b) For \( \frac{I_l}{I_t} > 200\% \)

<table>
<thead>
<tr>
<th>( t/\tau )</th>
<th>( \frac{\theta_C}{\theta_m} = \left( \frac{I_l}{I_t} \right)^2 \cdot t/\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_l/I_t = 2.5 )</td>
<td>( I_l/I_t = 3.0 )</td>
</tr>
<tr>
<td>0.02</td>
<td>0.125</td>
</tr>
<tr>
<td>0.04</td>
<td>0.25</td>
</tr>
<tr>
<td>0.06</td>
<td>0.375</td>
</tr>
<tr>
<td>0.08</td>
<td>0.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0.625</td>
</tr>
<tr>
<td>0.2</td>
<td>1.25</td>
</tr>
</tbody>
</table>

1 High values of \( \theta_C/\theta_m \) indicate that the motor can sustain such high currents for short durations only, i.e. at low \( t/\tau \).
2 These are danger areas and the machine must be prevented from operating in these areas as far as possible. If absolutely essential, the maximum permissible temperature may be exceeded for only a short period to protect the insulation from a rapid deterioration or damage (Section 9.2).
**For over-load settings**

$I_l/I_r$ versus $t/\tau$ different ratios of $\theta_c/\theta_m$ as shown in Table 3.2(a), and Figure 3.14(a) for $I_l/I_r \leq 200\%$ and Table 3.2(b) and Figure 3.14(b) for $I_l/I_r > 200\%$. Since $I_l$ and $\tau$ are known, $t$ can be calculated for different over-load conditions, corresponding to different temperature rises. The relay can then be set for optimum utilization of the machine.

**Example 3.2**

For an over-load of 25%, a class B motor, operating at an ambient temperature of 45°C, the relay corresponding to $I_l/I_r \leq 200\%$ can be set to trip as follows:

\[
\frac{\theta_c}{\theta_m} = 1.1 = (1.25)^2 (1 - e^{-t/\tau})
\]

or

\[
e^{-t/\tau} = 1 - \frac{1.1}{(1.25)^2}
\]

\[= 1 - 0.704\]

\[= 0.296\]

or

\[e^{t/\tau} = 3.378\]

\[
\therefore t = \tau \log_e 3.378
\]

\[= 1.5 \times 1.217 \text{ or } 1.82 \text{ hours}\]

Considering $\tau = 1.5 \text{ hours}$

The relay may therefore be set to trip in less than 1.82 hours.

**3.6.2 From hot conditions**

Similar curves can also be plotted for hot conditions using Equation (3.5) and assuming $\theta_0 = \theta_m$ for ease of plotting and to be on the safe side. The relay may then be set accordingly. For brevity these curves have not been plotted here.

One may appreciate that by employing a motor protection relay it is possible to achieve a near thermal image protection for all ratings, types and makes of motors through the same relay by setting its $I^2 - t$ and $\theta - t$ characteristics as close to the motor’s characteristics as possible. The O/C condition is normally detected through the motor’s actual heating, rather than current, for optimum utilization. Moreover, the starting heats or the heat of the previous running if it existed, when the motor was reswitched after a rest or a shutdown, are also accounted for by measuring the thermal content.

**Example 3.3**

The motor is operating hot, say at an end temperature of 130°C. If the motor is of insulation class B and ambient temperature at 50°C then,

\[\theta_c = 130 - 50 = 80°C\]

and \[\theta_m = 120 - 50 = 70°C\]

\[\therefore \frac{\theta_c}{\theta_m} = \frac{80}{70} = 114\%\]

Referring to the curves of Figure 3.13(a) the relay corresponding to an over-load of 150% will trip in about $t/\tau = 0.53$, for $\theta_c/\theta_m$ curve of 100% or 0.70 for $\theta_c/\theta_m$ curve of 120%.

If $\tau$ is taken as 1.5 hours, the motor may be set to trip in about $0.60 \times 1.5$ hours, i.e. about 54 minutes, considering the average value of $t/\tau$ as 0.60, for a $\theta_c/\theta_m$ as 114% by interpolation. It will be noted that a setting corresponding to a thermal curve of 100% will under-utilize while corresponding to 120% will overutilize the motor, while the optimum true

---

**Figure 3.14** Thermal curves to set the relay for over-temperature protection corresponding to different overload conditions

Note

For actual use combine curves $I_l/I_r \leq 200\%$ and $I_l/I_r > 200\%$ on one graph.
In terms of current settings ($I_t/I_r$ vs $I_t$)

(a) For $I_t/I_r \leq 200\%$, using the same equation

$$I_t = \frac{\theta_c}{\theta_m} \sqrt{\frac{1}{1 - \frac{1}{e^{t/\tau}}}}$$

<table>
<thead>
<tr>
<th>$t/\tau$</th>
<th>$1 - \frac{1}{e^{t/\tau}}$</th>
<th>$I_t/I_r$</th>
<th>$1 - \frac{1}{e^{t/\tau}}$</th>
<th>$I_t/I_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\theta_c/\theta_m = 0.1$</td>
<td>$\theta_c/\theta_m = 0.2$</td>
<td>$\theta_c/\theta_m = 0.5$</td>
</tr>
<tr>
<td>0.01</td>
<td>1 - $\frac{1}{1.01}$ = 0.0099</td>
<td>3.178</td>
<td>4.595</td>
<td>7.107</td>
</tr>
<tr>
<td>0.05</td>
<td>1 - $\frac{1}{1.05}$ = 0.0476</td>
<td>1.449</td>
<td>2.05</td>
<td>3.241</td>
</tr>
<tr>
<td>0.10</td>
<td>1 - $\frac{1}{1.105}$ = 0.095</td>
<td>1.026</td>
<td>1.451</td>
<td>2.294</td>
</tr>
<tr>
<td>0.20</td>
<td>1 - $\frac{1}{1.22}$ = 0.180</td>
<td>0.745</td>
<td>1.054</td>
<td>1.667</td>
</tr>
<tr>
<td>0.50</td>
<td>1 - $\frac{1}{1.65}$ = 0.394</td>
<td>0.504</td>
<td>0.712</td>
<td>1.126</td>
</tr>
<tr>
<td>1.0</td>
<td>1 - $\frac{1}{2.718}$ = 0.632</td>
<td>0.398</td>
<td>0.562</td>
<td>0.889</td>
</tr>
</tbody>
</table>

Notes:
1. For a closer overload protection, more curves should be drawn for $t/\tau > 1$.
2. *These points are not relevant for $I_t/I_r < 200\%$.

(b) $I_t/I_r > 200\%$, using Equation (b)

i.e. $I_t = \frac{\theta_c}{\theta_m} \sqrt{\frac{\tau}{t}}$

<table>
<thead>
<tr>
<th>$t/\tau$</th>
<th>$I_t/I_r = \frac{\theta_c}{\theta_m} \sqrt{\frac{\tau}{t}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_c/\theta_m = 0.1$</td>
</tr>
<tr>
<td>0.02</td>
<td>2.24</td>
</tr>
<tr>
<td>0.04</td>
<td>1.58</td>
</tr>
<tr>
<td>0.06</td>
<td>1.29</td>
</tr>
<tr>
<td>0.08</td>
<td>1.12</td>
</tr>
<tr>
<td>0.10</td>
<td>1.0</td>
</tr>
<tr>
<td>0.20</td>
<td>0.71</td>
</tr>
</tbody>
</table>

**These conditions may not occur even on a fault in the motor.

Note:
For obtaining a true replica of the motor thermal characteristics, $I^2 - t$ and $\theta - t$ more curves may be plotted for $t/\tau < 0.02$. 
utilization will correspond to $\theta_c/\theta_m$ as 114% only. It is therefore advisable to draw closer curves or use extrapolation whenever necessary to obtain a closer setting and plot a more true replica of the motor thermal characteristics.

### 3.7 Rating of short-time motors

If a short-time duty is performed on a Continuous Maximum Rating (CMR) motor with some no-load or idle running, the temperature rise $\theta$ may not reach its maximum value, $\theta_m$, as shown in curve (c) of Figure 3.11. A CMR motor therefore can be operated at higher outputs on short-time duties as shown in curve (d). The extent to which a CMR motor can be over-rated to perform a particular short-time or intermittent duty is considered in the following example. While evaluating the rating for such duties, the heat during start-up and during braking and their frequency of occurrence should be considered.

**Example 3.4**

(a) If a CMR 25 h.p. motor, with a thermal heating constant of 1.5 hours reaches a maximum temperature of 115°C in continuous operation with an ambient temperature of 40°C, then the half-hour rating $P$ of this motor can be determined as below.

- **Compare the temperature rises which are proportional to the losses at the two outputs and the losses are proportional to the square of the load.**
- **Ignoring the mechanical losses then**

$$\theta_m = \theta_m \left(1 - e^{-0.5 \times \frac{P}{25}}\right)$$

where $\theta_m = 115 - 40 = 75°C$

From (a) and (b)

$$\theta_m = \theta \left(\frac{P}{25}\right)^2$$

or $\theta_m = \theta_m \left(1 - e^{-0.5 \times \frac{P}{25}}\right) \cdot \left(\frac{P}{25}\right)^2$

or $1 = (1 - 0.716) \cdot \left(\frac{P}{25}\right)^2$

or $P = \frac{25}{\sqrt{0.284}}$

$= 47$ h.p.

(b) Similarly, if the rating is 1 hour, then

$$P = \frac{25}{\sqrt{1 - e^{-0.667}}}$$

$= \frac{25}{\sqrt{0.487}}$

$= 35.8$ h.p.

### 3.8 Equivalent output of short-time duties

For varying loads (Figure 3.15) or for short-time duties (Figure 3.16) it may not be necessary to select a motor corresponding to the maximum load during one cycle. Consider a motor that is always energized under the fluctuating loads of Figure 3.15. Then the equivalent requirement can be determined as below, ensuring that the output achieved and the motor chosen will be sufficient to develop a torque, during all conditions of voltages, adequate to drive even the highest load and meet its torque requirement. Consider heating to be proportional to the square of the loading, ignoring the mechanical losses. Then

$$P_{eq}(\text{r.m.s.}) = \sqrt{\frac{P_1^2 \cdot t_1 + P_2^2 \cdot t_2 + P_3^2 \cdot t_3}{t_1 + t_2 + t_3}}$$

(3.11)

Instead, if the load values represent the torque requirement, then

$$T_{eq}(\text{r.m.s.}) = \sqrt{\frac{T_1^2 \cdot t_1 + T_2^2 \cdot t_2 + T_3^2 \cdot t_3}{t_1 + t_2 + t_3}}$$

(3.12)

and motor output

---

**Figure 3.15** Equivalent output of short-time duties (varying loads)
If the cycle has a short-time rating, with a period of energization and one of rest, the motor will obviously cool down during the de-energizing period, and depending upon the peak load and the rest periods, a comparatively lower output motor can perform duties at higher loads.

The equivalent output for the load cycle of Figure 3.16 is

\[ P_{eq} = \frac{P_1 \cdot t_1 + P_2 \cdot t_2 + P_3 \cdot t_3}{t} \]

Since total time \( t \) is more in this instance, the equivalent power required will be less.

**Example 3.5**
Determine the motor rating, for a 10-minute cycle operation as shown in Figure 3.17. There is no rest period, but in one cycle, the motor runs idle twice at no-load for 2 minutes each. The cycle starts with a load requirement of 10.5 kW for 4 minutes followed by an idle running, a load of 7.5 kW for 2 minutes, again with an idle running and then the cycle repeats.

**Solution**
Assuming the no-load losses to be roughly 5% of the motor rating of, say, 10 kW, then

\[ P_{eq} = \sqrt{\frac{(10.5)^2 \times 4 + (0.5)^2 \times 2 + (7.5)^2 \times 2 + (0.5)^2 \times 2}{10}} \]

\[ = \sqrt{\frac{441 + 0.50 + 112.50 + 0.50}{10}} \]

\[ = \sqrt{55.45} \]

\[ = 7.45 \text{ kW} \]

The nearest standard rating to this is 7.5 kW, and a motor of this rating will suit the duty cycle. To ensure that it can also meet the torque requirement of 10.5 kW, it should have a minimum pull-out torque of 10.5/7.5 or 140% with the slip at this point as low as possible so that when operating at 140% on its speed–torque curve, the motor will not drop its speed substantially to cause high slip losses.

**Note**
For such duties, the starting heat is kept as low as possible by suitable rotor design to eliminate the effect of frequent
starts and stops. The margin for starting heat and braking heat should be taken into account if these are considerable. The manufacturer is a better guide for suggestions here.

### 3.9 Shock loading and use of a flywheel

The application of a sudden load on the motor for a short duration, in the process of performing a certain load duty, is termed ‘shock loading’. This must be taken into account when selecting the size of a motor. Electric hammers, piston pumps, rolling mills, cane crushers and cane levellers, sheet punching, notching, bending and cutting operations on a power press, a brake press or a shearing machine are a few examples of shock loading. They all exert a sudden load, although for a very short duration, during each load cycle, and may damage the motor as well as the machine. Such machines, therefore, experience a sharp rise and fall in load. Figure 3.18 depicts such a load cycle, having excessive load $P_2$ for a short duration $t_2$, a very light load $P_1$ for a duration $t_1$ and at no-load for rest of one cycle.

For such load requirements, one may either choose a comparatively larger motor to sustain the load and torque requirements during shock loading or a smaller motor,
depending upon the average equivalent loading \( P_{eq} \) as discussed earlier. When choosing a smaller motor it would be advisable to absorb and smooth the shocks first to contain the additional shock burden on the motor, as well as on the main machine. This is made possible by adding more moments of inertia to the drive by introducing a flywheel in the system, as shown in Figure 3.19. The flywheel will now share a substantial jerk of the peak load, because it possesses a high inertia, on the one hand, and is already in motion, on the other, when the load jerk is applied. The motor now has to share only a moderate jerk and a smaller motor can safely perform the required shock duty. During peak load, the stored kinetic energy of the flywheel is utilized to perform the load requirement. This energy is regained when the motor picks up after performing the task. Motors for such applications can be built with larger air gaps which may mean a low power factor and a higher slip, but a higher capacity to sustain shocks.

### 3.9.1 Size of flywheel

This is a mechanical subject, but is discussed briefly for more clarity. The size of the flywheel, as well as the size of the motor, will depend upon the speed variation that will be permissible for the type of duty being performed. It should be such that by the time the machine is required to perform the next operation it has gained enough momentum and regained its consumed energy capable of performing the next operation without undue stress on the motor. This permissible speed variation may be as low as 1–2% in steam engines and as high as 15–20% for punches and shears, etc.

### 3.9.2 Energy stored by the flywheel

\[
F = \frac{W \cdot V_1^2}{2 \cdot g} \text{ Joules} \tag{3.13}
\]

where

- \( F \) = energy stored by the flywheel in Joules
- \( W \) = weight of the flywheel in kg
- \( V_1 \) = velocity of the flywheel in m/s
- \( g \) = 9.81 m/s\(^2\)

After performing the duty, if the velocity of the flywheel drops to \( V_2 \) then the energy shared by the flywheel while absorbing the shock load

\[
= \frac{W(V_1^2 - V_2^2)}{2 \cdot g} \text{ Joules}\]

From the peak load \( P_2 \) and from the available h.p. of the motor \( P_{eq} \) we can determine the energy to be shared by the flywheel, i.e.

\[
T_2 - T_{eq} = \frac{W(V_1^2 - V_2^2)}{2 \cdot g} \tag{3.14}
\]

\((T_2 \text{ and } T_{eq} \text{ are in Joules})\)

From this one will be able to ascertain the weight of the flywheel in kg. The velocity \( V \) of the flywheel is a design parameter of the basic machine and is derived from there. Based on the speed of the flywheel and weight \( W \), the diameter and width and other parameters, as required to design a flywheel, Figure 3.20 can be easily determined with the help of any mechanical engineering handbook.

---

**Figure 3.19** A brake press illustrating the use of a flywheel (Courtesy: Prem Engineering Works)

**Figure 3.20** Flywheel
**Duties of induction motors**

### List of formulae used

#### Factor of inertia

\[
FI = \frac{GD_M^2 + GD_L^2}{GD_M^2} \quad (3.1)
\]

\[GD_M^2 = \text{M.I. of motor}\]

\[GD_L^2 = \text{M.I. of load at motor speed}\]

\[\text{M.I.} = \text{moment of interia}\]

#### Heating curves

**Exponential heating on a cold start**

\[
\theta_c = \theta_m (1 - e^{-t/\tau}) \quad (3.2)
\]

\[\theta_c = \text{temperature rise on a cold start above } \theta_a \text{ after } t \text{ hours in } ^\circ\text{C.}\]

\[\theta_a = \text{end temperature of the machine in } ^\circ\text{C after time } t\]

\[\theta_m = \text{ambient temperature in } ^\circ\text{C}\]

\[t = \text{tripping time of the relay in hours}\]

\[\tau = \text{heating or thermal time constant in hours}\]

\[K \cdot I^2_t = I^2_t (1 - e^{-t/\tau}) \quad (3.3)\]

\[I_t = \text{rated current of the motor in A}\]

\[K = \text{a factor that depends upon the type of relay (generally 1 to 1.2)}\]

\[I_1 = \text{actual current}\]

**Exponential heating on a hot start**

\[
\theta_h = I^2_0 (1 - e^{-t/\tau}) \quad (3.4)
\]

\[\theta_h = \text{temperature rise on a hot start above } \theta_a \text{ after } t \text{ hour in } ^\circ\text{C.}\]

\[\theta_a = \theta_c - \theta_h\]

\[I_0 = \text{initial current at which the machine was operating}\]

\[I_1 = \text{actual current of the machine}\]

\[t = \tau \log_e \frac{I^2_0 - I^2_t}{I^2_1 - k I^2_t} \quad (3.5)\]

**Adiabatic heating on a cold start**

\[
\theta_c = \theta_m - \theta_a = I^2_t \cdot \frac{t}{\tau} \quad (3.6)
\]

**Adiabatic heating on a hot start**

\[
\theta_h = \theta_c - \theta_a = I^2_0 + (I^2_t - I^2_0) t/\tau \quad (3.7)
\]

**Cooling curves**

**Exponential temperature fall when } I_t = 0**

\[
\theta = \theta_m \cdot e^{-t/\tau'} \quad (3.8)
\]

\[\tau' = \text{cooling time constant in hours}\]
To draw the thermal curves

From cold conditions

(a) When \( I_t \leq 200\% \ I_r \)

\[
\frac{\theta_c}{\theta_m} = \left( \frac{I_t}{I_r} \right)^2 \left( 1 - \frac{1}{e^{I_t/2}} \right)
\]  
(3.9)

(b) When \( I_t > 200\% \)

\[
\frac{\theta_c}{\theta_m} = \left( \frac{I_t}{I_r} \right)^2 \cdot \frac{t}{\tau}
\]  
(3.10)

Equivalent output of short-time duties

\[
P_{eq\ (r.m.s.)} = \sqrt{\frac{P_1^2 \cdot t_1 + P_2^2 \cdot t_2 + P_3^2 \cdot t_3}{t_1 + t_2 + t_3}}
\]  
(3.11)

\[
T_{eq\ (r.m.s.)} = \sqrt{\frac{T_1^2 \cdot t_1 + T_2^2 \cdot t_2 + T_3^2 \cdot t_3}{t_1 + t_2 + t_3}}
\]  
(3.12)

\[
P_{eq} = \frac{T_{eq} \cdot N_r}{974} \text{ kW}
\]

Shock loading

Energy stored by the flywheel

\[
F = \frac{W \cdot V_1^2}{2 \cdot g} \text{ Joules}
\]  
(3.13)

\( F \) = energy stored by the flywheel in Joules
\( W \) = weight of the flywheel in kg
\( V_1 \) = velocity of the flywheel in m/s
\( g = 9.81 \text{ m/s}^2 \)

Energy to be shared by the flywheel

\[
T_2 - T_{eq} = \frac{W(V_1^2 - V_2^2)}{2 \cdot g}
\]  
(3.14)

\( T_2 \) = corresponding to peak load \( P_2 \)
\( T_{eq} \) = corresponding to the h.p. of the motor